Title: Systems of Linear Equations – Elimination (Addition) Method (Part 2)

Class: Math 100 or Math 107

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Instructions to tutor: Read instructions and follow all steps for each problem exactly as given.

Keywords/Tags: systems, systems of linear equations, elimination, addition, consistent, inconsistent, dependent

<u>**Systems of Linear Equations – Elimination (Addition) Method**</u>

<u>**Purpose:**</u>

This is intended to refresh your knowledge about solving systems of linear equations using the elimination (addition) method, along with analyzing the solution.

Recall that a **system of equations** consists of two or more equations each with two or more variables. A **solution** to a system in two variables is an ordered pair (x, y) that satisfies each equation in the system. For now, we will concentrate on systems of linear equations.

Substitution Method – Solve for one of the variables in one of the equations and substitute it into the other equation. After this is done, you will have a single equation with one variable – solve for it. Then back-substitute to find the other.

Elimination (Addition) Method – Add a multiple of one equation to the other in order to eliminate one of the variables. After this is done, you will have a single equation with one variable – solve for it. Then back-substitute to find the other.

It is important to note that not all systems have a unique solution – so we need to consider all possibilities when working toward a solution. A system of two linear equations in two variables can have one solution, no solution, or an infinite number of solutions:
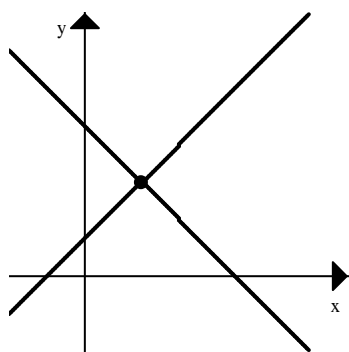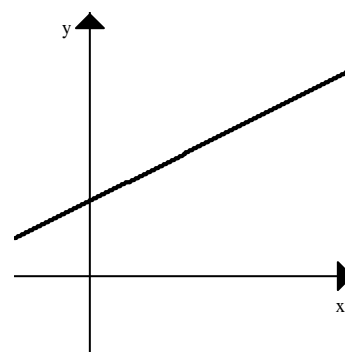

Consistent System
Intersecting lines – One Solution
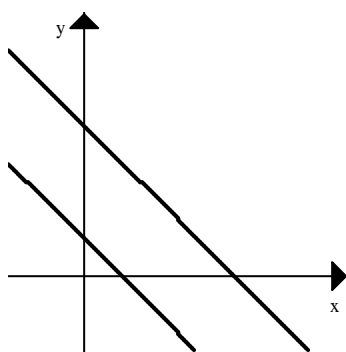
Inconsistent System
Parallel lines – No Solution

Dependent System
Same line – Infinite Solutions

You have seen that if you have a consistent system, you get a single solution. So what happens algebraically when there is no solution or when there are an infinite number of solutions? We will apply the elimination method to the following systems.

Example: $\begin{cases} 2x - 5y = 7 \\ -4x + 10y = 2 \end{cases}$ Let's add twice the 1$^{\text{st}}$ equation to the 2$^{\text{nd}}$ to eliminate x.

$$\begin{array}{r} 2(\ 2x - 5y = 7) \\ + \quad -4x + 10y = 2 \\ \hline 0 = 16 \end{array}$$

Do you see anything that bothers you about the equation we obtained? Of course zero does not equal 16! So our assumption that the system had a solution moved us to a contradiction. This means our assumption was incorrect, and the system has no solution. This means that the two lines are parallel and the system is *inconsistent*.

Example: $\begin{cases} 2x - 5y = 7 \\ -4x + 10y = -14 \end{cases}$ We again will add twice the 1$^{\text{st}}$ equation to the 2$^{\text{nd}}$ to eliminate x.

$$\begin{array}{r} 2(\ 2x - 5y = \quad 7) \\ + \quad -4x + 10y = -14 \\ \hline 0 = 0 \end{array}$$

So what bothers you about the equation obtained this time? Nothing! But, we are no closer to obtaining a solution. What happened is that we eliminated an equation with the other. This means that one equation was a multiple of the other – this means that they are really two equations for the same line. So we have an infinite number of solutions.

How do we write them all down? We do not have enough time to write down an infinite number of ordered pairs, so we solve one of the equations for one of the variables. This gives us a formula representing all solutions to the system.

Solving the 1$^{\text{st}}$ equation for y, we obtain $y = \frac{2}{5}x - \frac{7}{5}$. Note that this is an equation of y depending on x – this is why we call it a *dependent* system.

Our solution is given by the ordered pairs of the form $\left(x, \frac{2}{5}x - \frac{7}{5}\right)$. So for any value you choose for x, you will obtain a solution to the system. Can you list 3 solutions?

Example: Now solve each of the following systems, identifying them as *consistent*, *dependent*, or *inconsistent*. For the consistent systems, give a solution. For the dependent systems, give a general solution and 3 particular solutions.

(a) $\begin{cases} -2x + 3y = 10 \\ 8x - 12y = 4 \end{cases}$

(b) $\begin{cases} \frac{3}{2}x - 5y = 1 \\ x + \frac{3}{4}y = -1 \end{cases}$ (Hint: First try clearing both equations of fractions.)

(c) $\begin{cases} 3x - y = 5 \\ -6x + 2y = 10 \end{cases}$

Check your answers – If you did not get these, consult a tutor for help.

(a) No solution, inconsistent

(b) Consistent, $\left(\frac{-34}{49}, \frac{-20}{49} \right)$

(c) Dependent, $(x, 3x - 5)$; $(0, -5)$, $(1, -2)$, $(-5, -20)$