Properties of the Real Numbers

The following are the properties of addition and multiplication if x, y, and z are real numbers:

	Addition	**Multiplication**
Commutative	$x+y=y+x$	$x\cdot y=y\cdot x$
Associative	$(x+y)+z=x+(y+z)$	$(x\cdot y)\cdot z=x\cdot(y\cdot z)$
Identity	$x+0=x$	$x\cdot 1=x$
Inverse	There is a unique number $-x$ such that $x+(-x)=0$	If $x\neq 0$, there is a unique number $\frac{1}{x}$ such that $x\cdot\frac{1}{x}=1$

Distributive	$x\cdot(y+z)=x\cdot y+x\cdot z$
Multiplication by zero	$x\cdot 0=0$

Commutative Property: When adding or multiplying *two* numbers, the order of the numbers can be reversed without changing the result.

Addition: $3+5=5+3$ now check! $3+5=$___ and $5+3=$___

Multiplication: $4\cdot 7=7\cdot 4$ now check! $4\cdot 7=$___ and $7\cdot 4=$___

Associative: When adding or multiplying *three or more* numbers, the result does not change if the numbers are grouped differently.

Addition: $(1+2)+3=1+(2+3)$ now check!

$(1+2)+3=($____$)+3=$___ and $1+(2+3)=1+($____$)=$___

Multiplication: $(1\cdot 2)\cdot 3=1\cdot(2\cdot 3)$ now check!

$(1\cdot 2)\cdot 3=($___$)\cdot 3=$___ and $1\cdot(2\cdot 3)=1\cdot($___$)=$___

Identity: Addition and multiplication each have an *identity element*. This is a special number that does not change the value of other numbers when combined. For addition this number is *zero*, and for multiplication the number is *one.*

Addition: $5+0=$___

Multiplication: $5\cdot 1=$___

Inverse: Addition and multiplication each have a unique *inverse element* for each real number (except *zero* for multiplication!) A number combined with its *inverse* gives the *identity element*.

Addition: $5+(-5)=$___

Multiplication: $5\cdot\frac{1}{5}=$___

Distributive: We say that multiplication *distributes* over addition of real numbers.

$2\cdot(1+3)=2\cdot 1+2\cdot 3$ now check! $2\cdot(1+3)=2\cdot($____$)=$___ and $2\cdot 1+2\cdot 3=$___$+$___$=$___

<u>Addition does not distribute over multiplication!</u>

$2+(1\cdot 3)\neq(2+1)\cdot(2+3)$ because $2+(1\cdot 3)=6$ and $(2+1)\cdot(2+3)=15$

Multiplication by zero: Any real number multiplied by zero is equal to zero.

$5\cdot 0=$___